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CONSIDERING TUNING: FROM BACH TO 21ST CENTURY TRENDS

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A THESIS SUBMITTED TO

THE FACULTY OF GRADUATE STUDIES

IN PARTIAL FULFILMENT OF THE REQUIREMENTS

FOR THE DEGREE OF

MASTER OF ARTS

GRADUATE PROGRAM IN MUSIC

YORK UNIVERSITY,

TORONTO, ONTARIO

JUNE 2017

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ABSTRACT

This thesis is an exploration of instrumental tuning trends from Bach to 21st Century, considering different areas of musicological and acoustical studies. While the introduction brings us in a tuning continuum with brief explanations from the origins, the first chapter details the controversial acoustic theories of Hermann Helmholtz and Bernhard Riemann over various tuning temperaments. Considering baroque temperaments having a contoured profile in between all historical tunings, the present work builds the special interest for the most influential schemata, which impacted classic and contemporary tunings.

The second chapter makes a case for Werckmeister's well-tempered tunings and the mathematical approaches of historical temperaments during Bach's time.

The third chapter details the development of the equal tempered tuning and the evolution from the meantone to classic or equal temperament
I plan to have a closer look into geometrical and mechanical approximations that led to equal divisions.

The fourth chapter explores the immense spectrum of micro tunings and digital technologies. At this point I want to underline the rapid ascension of digital technologies emphasizing the latest discoveries of sound emulation

methods and sample image engendering, drawing the line to inferring from evidence and reasoning how the temperaments of the future might develop.

ACKNOWLEDGEMENTS

I would like to express my thanks especially to my supervisor Michael Coghlan and my committee member Stephanie Martin. Your encouragement and constructive leadership along the last two years were a real "positive reinforcement" to my education development. I am so grateful for your decision to supervise my work and I hope that I succeeded to reach the level expected of me. Your suggestions and advice on the practical and theoretical related aspects of this thesis, everything on the premises of your extensive experience as accomplished musicians and university professors, were decisive in helping me explore the musicology stream I am passionate about, acoustics.

Similarly, I would also like to thank York music librarian Rob van Der Bliek for being my right hand in finding the right information, in the right place, and at the right time. Your comments and kind suggestions played an important role in helping me pursue this research and complete my thesis.

Mrs. Teresa Tilban-Rios deserves a special mention for her expertise and professionalism. Thank you so much for your calm, patient, and detailed help with the organizational aspects of our activity. Your patience and "endurance" during this long process assured a good fundament for all students to be successful.

Also, my thanks to all my colleagues and the participants in this research.

A big part of what I accomplished here, is because of really good friends like you!

Thank you all!

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CHAPTER I

INTRODUCTION

Music is a hidden arithmetical exercise of the soul who is unaware that he/she is counting.

(Gottfried Leibnitz, *Théodicée*, Preface, 1710)

The tuning of musical instruments most probably developed concomitantly with the creation of the instrument itself, or perhaps, it is as ancient as the first attempts of forming a musical structures. Despite this reality, many people believe that the concern with instrument tuning represents a fairly recent preoccupation fostered by the enlightenment era and the industrial progress. There appears to be a generalized opinion that a loss of relevance regarding historical tunings occurred after the classical music period, or more immediately after the ascension of equal temperament. Barbour states

> if primitive man played upon an equally primitive instrument only two different pitches, these would represent an interval of some sort — a major, minor, or neutral third; some variety of fourth or fifth; a pure or impure octave. Perhaps his concern was not with interval as such, but with the spacing of sound holes on a flute or oboe, the varied lengths of the strings on a lyre or harp.
>
> (Murray J. Barbour, *Tuning and Temperament, a historical survey*, 1951, pg.1)

Instrument tuning marks an important step up in human civilization being one of the first endeavors to incipient universal culture, and on the other hand, historical tunings represent the crucial ascendant level for understanding the basic principles of tuning, and a major development towards the equal temperament.

My purpose in this paper is: 1) to present a brief historical tunings chronology and brief description detailing of some major theoretical aspects; 2) to consider how “alternate” tunings might influence contemporary instrumental performance, and to what extent they could still be relevant in a very advanced musical world with respect to the technical aspects of instrument construction, recording, performing and sound engineering; 3) after grasping a good sense tuning evolution, to infer from evidence and reasoning, how alternate tunings might develop and evolve in the future.

1.1 Historical tunings

1.1.1 Pythagoras

A brief chronological description of the tuning history must consider Pythagoras first. Before the invention of meantone tuning, which was the main tuning schemata in Middle Ages, the French school of polyphony at Notre Dame (13th and 14th centuries) followed an early medieval convention since Boethius (4th century AD) in declaring that only a series of perfect or pure fifths could generate a scale. Pythagoras innovations and his mathematical system of tuning has had a profound influence upon both the antiquity and the modern world.

Nowadays, having a broad picture of the tuning phenomenon, we can affirm that a great many irregular temperaments were largely based on Pythagorean in that they contain many pure fifths. These are not especially difficult to tune. Considered in relation to the slightly narrow fifths of equal temperament, the pure fifths are the first stage in the process of tuning a tempered instrument[1].

> The Pythagorean temperament is based upon the octave and the fifth, the first two intervals of the harmonic series. Using the ratios of 2:1 for the octave and 3:2 for the fifth, it is possible to tune all the notes of the diatonic scale in a succession of fifths and octaves, or, for that matter, all the notes of the chromatic scale. Thus, a simple, but rigid, mathematical principle underlies the Pythagorean tuning.
>
> (Murray J. Barbour, *Tuning and Temperament, Introduction*, 1951, pg.1)

Consequently, it is not only one of the easiest to tune by ear, but also it is based on a mathematically demonstrations of simple ratios. Pythagorean tuning determines all notes and intervals of a scale starting with the circle of pure on untempered fifths, with a ratio of exactly 3:2. To acquire a complete chromatic scale of the kind common on keyboards as early as 13th century, a series of 11 perfect fifths should be displayed where the originating point is middle D:

Table 1: Pythagorean pure fifths

Eb	Bb	F	C	G	**D**	A	E	B	F#	C#	G#

[1] When a tempered instrument (piano, keyboard, organ) is tuned, firstly the fifths are made pure (no acoustic beats), and secondly, they are narrowed by an equal part of the Pythagorean comma.

Tuning a series of pure fifths reveals the one potential pitfall of this system, in that the fourth or fifth between the extreme notes of the series, Eb-Ab, an extremely out of tune interval. This fifth, called in the plastic and colourful language of intonation, the "wolf" interval, has strong acoustic beats which simply cannot be ignored. The explanation is based on the fact that 12 perfect fifths do not round off to precisely a pure octave interval, but exceed it by a small acoustic difference known as the Pythagorean comma[2].

Another stage up trying to understand the implications of an un-equal tuning, is the frequency of using a “wolf interval” in practical music. Gracefully, the last fifth of the circle in Pythagorean tuning, the Eb-G# (or enharmonically Ab) was rarely used together as a blocked interval in early medieval harmony, either because it was purposely avoided, or the development stage of music theory was still in progress. Even both reasons might be taken into consideration simply because in that period of music history, this was hardly a practical problem.

All intervals have small integer ratios[3] based on the powers of two and three. The following table shows the Pythagorean tuning being a just-intonation

[2] **Pythagorean comma** is the small interval existing in Pythagorean tuning between two enharmonically equivalent notes such as C and B♯, or D♭ and C♯. The fractional ratio it is equal to the frequency ratio 531441:524288, or approximately 23.46 cents, roughly a quarter of a semitone (in between 75:74 and 74:73). This is the comma which musical temperaments often refer to as the one to temper, the Pythagorean comma. The Pythagorean comma can be also defined as the difference between a chromatic and a diatonic semitone, as determined in Pythagorean tuning, or the difference between twelve just perfect fifths (3/2) and seven octaves (2/1).

[3] **Whole number ratios** can be expressed by small-integer ratios, such as 1:1 (unison), 2:1 (octave), 3:2 (perfect fifth), 4:3 (perfect fourth), 5:4 (major third), minor seventh (16:9). In tuning and temperament, those intervals with small-integer ratios are called just intervals, or pure intervals.

scale on a series of perfect fifths, all the ratio numbers powers of either two or three:

Table 2: Ratio numbers powers of either two or three

Pitch:	C	C#	D	Eb	E	F	F#	G	G#	A	A#	B	C
Ratios	1/1	2187/2048	9/8	32/27	81/64	4/3	729/512	3/2	128/81	27/16	16/9	243/128	2/1
Cents:	0	113.7	203.9	294.1	407.8	498	611.7	702	792.2	905.9	996.1	1109.8	1200

From that point of view, the Pythagorean tuning is a form of just intonation based on the numbers three and nine. In fact,

> Pythagorean tuning is described in the medieval sources as being based on four numbers: 12:9:8:6. Jacobus of Liege (c. 1325) describes a "quadrichord" with four strings having these lengths: we get an octave (12:6) between the outer notes, two fifths (12:8, 9:6), two fourths (12:9, 8:6), and a *tonus* or major second between the two middle notes (9:8).
> (from www.medieval.org/Pythagorean tuning/Basic concepts - http://www.medieval.org/emfaq/harmony/pyth2.html)

This scale was relevant for a musical period in which perfect fifths, fourths and octaves were the dominant sonority, and in which the other intervals like thirds were theoretically considered dissonances and consequently avoided at final cadences. Taking into consideration the austere sonorities created by using mostly perfect intervals, it is easy to understand why pitches like C#, F#, and G#

appeared rarely, and in relation to other "pure pitches" they even received a certain mystical connotation.[4]

The remaining "room" in the octave leads to other intervals that could be subtracted from these basic intervals, revealing in an early medieval music background, an explicit image of interval ratios with both practical and theoretical aspects. The differences between primary intervals and the octave procues the key for calculating the rest of intervals.

The following table demonstrates the relation between standard intervals of Pythagorean tuning except the pure unison (1:1) and octave (2:1), and the mathematical calculated perfect ratios. Those intervals are derived primarily from the circle of pure fifths (3:2), thus having ratios which are powers of 3:2. Here follows chart of the 13 common intervals of medieval music from unison to octave as listed by Anonymous I around 1290, and by Jacobus of Liege in 1325.

[4] The tritone interval surnamed "Diabolus in musica" is a restless interval, classed as a perfect dissonance in the history of music from the early Middle Ages. It was treated with caution and frequently avoided in medieval singing due to its dissonant qualities.

Table 3: 13 usual intervals of medieval music

Interval	Ratio	Derivation	Cents
Unison	1:1	Unison 1:1	0.00
Minor Second	256:243	Octave - M7	90.22
Major Second	9:8	(3:2)^2	203.91
Minor Third	32:27	Octave - M6	294.13
Major Third	81:64	(3:2)^4	407.82
Fourth	4:3	Octave - 5	498.04
Augmented Fourth	729:512	(3:2)^6	611.73
Fifth	3:2	(3:2)^1	701.96
Minor Sixth	128:81	Octave - M3	792.18
Major Sixth	27:16	(3:2)^3	905.87
Minor Seventh	16:9	Octave - M2	996.09
Major Seventh	243:128	(3:2)^5	1109.78
Octave	2:1	Octave 2:1	1200.00

(from www.medieval.org/Pythagorean tuning/Basic concepts - http://www.medieval.org/emfaq/harmony/pyth2.html)

In Pythagorean tuning the major thirds are a ditonic comma[5] (about 1/9 tone) sharper than the pure thirds of the harmonic series.

> The wide thirds do provide a compelling pull to the perfect fifths they usually resolve outward to; that is, in a cadence typical of Guillaume de Machaut (c. 1300-1377), a D and F# 408 cents apart will move outwardly to C and G.
>
> (Gann, Kyle, *An Introduction to historical tunings* – http://www.kylegann.com/histune.html)

[5] A **ditone** (or major third) is an interval formed by two major tones. In Pythagorean tuning, a major tone has a size of about 203.9 cents (frequency ratio 9:8), thus a Pythagorean ditone is about 407.8 cents.

1.1.2 Ditonic and syntonic comma

> The foundation of the Greek scale was the tetrachord, a descending series of four notes in the compass of the modern perfect fourth. Most typical was the Dorian tetrachord, with two tones and then a semitone, as A G F E or E D C B. Two or more tetrachords could be combined by conjunction, as the above tetrachords would be with E a common note. Or they might be combined by disjunction, as the above tetrachords would be in reverse order, with a whole tone between B and A. Tetrachord combined alternately by conjunction and by disjunction correspond to our natural heptatonic scale.
>
> (Murray J. Barbour, *Tuning and Temperament, Greek tunings*, 1951, Pg.15)

Greek music theory had three tuning genres—diatonic, chromatic, and enharmonic, which are important for their influence on, and contribution to, modern tunings. It is appropriate to consider Greek tetrachords since the main consideration in interval tuning was the tetrachord and not the octave. A tetrachord contains three intervals and four notes. The term literally means four strings, a clear reference to instruments like harp, lyre, or kithara. The main distinction between these three genera was defined by the largest of the three intervals of the tetrachord.

A diatonic tetrachord contained two tones and a semitone variously arranged, where the tone (approx. 200 cents in size) is the mark of the main and most common genera. For the chromatic tetrachord, the characteristic interval is the minor third with a size of approximately 300 cents. The enharmonic tetrachord displays the characteristic interval of the ditone or the major third.

> In the chromatic tetrachord, the second string (as G) was lowered until the two lower intervals in the tetrachord were equal. Thus, A G" F E represents the process of formation better than the more commonly shown A F# F E. In the enharmonic tetrachord the second string was lowered still further until it was in unison with the third string; the third string was then tuned half way between the second and fourth strings.
>
> (Murray J. Barbour, *Tuning and Temperament*, *Greek tunings*, 1951, Pg.15)

From the theorists of antiquity, Claudius Ptolemy presented the most complete list of tunings, and advocated for them himself.[6] His field research suggested three enharmonic, eight chromatic, and ten diatonic tunings. Modern music systems have been influenced by only two of these seventeen or eighteen independent tunings - the third and fourth of Ptolemy's diatonic scales, commonly called the "ditonic" and the "syntonic."[7] As a matter of fact, the first one, the "ditonic" or "diatonic "(similar to Eratosthenes' diatonic), in reality is the Pythagorean tuning, with the pair of equal tones forming a major third (ditone) setting its characteristic name. The second one has a slightly larger stretch and is the temperament known today as Just intonation.[8]

[6] Claudii Ptolemaei Harmonicorum libri tres . Latin translation by John Wallis (London, 1699).

[7] The syntonic comma, also known as the chromatic diesis, the comma of Didymus, the Ptolemaic comma, is a small comma type interval between two musical notes, equal to the frequency ratio 81:80 (around 21.51 cents). The comma is referred to as a "comma of Didymus" because it is the amount by which Didymus corrected the Pythagorean major third (81:64, around 407.82 cents) to a just major third (5:4, around 386.31 cents).

[8] The Pythagorean comma existing in Pythagorean tuning between two enharmonically equivalent notes such as C and B♯ it is equal to the frequency ratio 531441:524288, or approximately 23.46 cents, roughly a quarter of a semitone. The comma which musical temperaments often refer to tempering is the Pythagorean comma, which can be also defined as the difference between a chromatic and a diatonic semitone, as determined in Pythagorean tuning.

With a piano keyboard tuned in equal temperament, a group of four tempered fifths (700 x 4 = 2800 cents) is exactly equal to two octaves (1200 x 2 = 2400 cents) plus a major third (400 cents). Either variant we choose, starting from a C, both combinations of intervals will finish at E. However, using justly tuned octaves (2:1), fifths (3:2), and thirds (5:4) with no acoustic beats, will result in two slightly different pitches with the ratio of (81:80) between their frequencies. That very small interval called the syntonic comma (21.51 cents), is the most important of the musical commas for the notation of musical intervals.

> While the ditonic or pythagorean comma aids in the construction and analysis of tuning systems, the Syntonic comma defines the way thirds are notated in chords. The syntonic comma is defined as the difference between a pythagorean major third (made from the difference twixt 4 just 3/2 perfect fifths up and 3 octaves) and a 5/4 major third.
>
> Brian McLaren, *Encyclopedia of microtonal music theory*, http://www.tonalsoft.com/enc/s/syntonic-comma.aspx

1.1.3 Aristoxenus

Another seminal figure in the history of Greek tunings is Aristoxenus. His theory opposed the opinion of Pythagoras' disciples that arithmetic rules were considered the ultimate judge for intervals and temperaments. The general idea of antics that in every system there must be found a mathematical coincidence before such a system can be said to be harmonic, influenced centuries of theoreticians after that. Aristoxenus tried to find the answer to an essential question: Are the mathematical calculations of theorists as important as the observations of musicians themselves?

From very early in human history the abstract concepts of mathematics would be perceived as different...or "too" pure in comparison with what human ear eventually would admit to be logical.

> In his second book Aristoxenus asserted that by the hearing we judge of the magnitude of an interval, and by the understanding we consider its many powers. The nature of melody is best discovered by the perception of sense, and is retained by memory; there is no other way of arriving at the knowledge of music.
> (Sir John Hawkins, General history of the science and practice of music, Chapter XIV, Pg.67, 1868)

Aristoxenus' protest was focused mainly against the rigidity of mathematical theories, and his specific conclusion was that the judgment of the "musical" ear with regard to intervals was superior to mathematical ratios. From the whole list of seven temperaments or scales proposed, he has one which is composed of equal tones and equal halves of tones, fact that influenced the sixteenth century theorists to sustain that he was in fact, the inventor of equal temperament.[9]

1.1.4 Claudius Ptolemy

After these two antagonistic theories which had been more or less seeking mathematical proofs, there was the third great figure in early tuning history,

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[9] 'The fourth is two and a half tones' is exactly what Aristoxenus states at several places. The geometrical methods instead of arithmetical methods, were the way he explained to his contemporaries about the harmonic relationships as ratios, which is closer to the equal temperament

Claudius Ptolemy**.** For Ptolrmy the main concern was the agreement, or the common ground, between music and mathematics. This was the right movement for that generation, an excellent principle in tuning knowledge: the tuning is best for which ear and ratio are in agreement. He advocated for understanding, and he claimed that it is possible to reach harmony between mathematicians and musicians. The need to announce the results and help other sciences to reach their goals, probably would be a result of a compromise on both sides. This new "friendly way" of different sciences of explaining the nature around us, was on the opposite side with the aggressive and exclusivist methods.

> To Ptolemy the matter was much simpler, a tuning was correct if it involves tetrachords and octaves and used super-particular ratios, such as 5:4, 11:10, etc., not to relate mathematics of music only with the specific ratio of 3:2 and 2:1 like in Pythagorean tuning. Ptolemy's syntonic diatonic has especial importance to the modern world because it coincides with just intonation, system founded on the first five intervals of the harmonic series—octave, fifth, fourth, major third, minor third.
>
> (Murray J. Barbour, *History of tuning and temperament*, 1951, Pg.2)

1.1.5 Didymus

The "comma of Didymus" named after him, was the amount by which he modified the size of the Pythagorean major third (81:64, around 407.82 cents) to a just major third (5:4, around 386.31 cents). From his proposed temperaments, Didymus' diatonic used the same ratios for his intervals, but in slightly different structure. The antiquity schemata of just intonation, in either versions by Ptolemy or Didymus, though unknown throughout the Middle Ages, was the aesthetic ideal of the Renaissance theorists.

The just intonation temperaments of early medieval music were “comfortable” for the unisonal Gregorian chant, because its small semitones are excellent for melody composition and its sharp major thirds sounded satisfactory. When the first incipient attempts at “harmony” produced the parallel octaves, fourths and fifths of organum, Pythagorean tuning was secure continued in use for many centuries.

Starting with Cantus Gemellus, thirds and sixths were freely used and they were considered imperfect consonances rather than dissonances based on the process of tempering. Didymus questioned whether these thirds and sixths were as rough as they would have been in the strict Pythagorean tuning, and he begun the process of softening or tempering of the Pythagorean major thirds. The just tuning practice influenced many ancient musical cultures and is still used in our modern times.

1.2 Meantone Temperament

1.2.1 Gioseffo Zarlino and Francisco de Salinas

A very interesting study is represented by the history of the meantone temperament, since various theorists in addition to Gioseffo Zarlino and Francisco de Salinas had contradictory ideas as to the amount by which the fifths should be tempered in order to save the purity of thirds. Another name to be mentioned here is Silbermann, the baroque famous keyboard instruments builder. His temperament of 1/6 comma for the fifths is the most important for modernity, because it corresponds to the more conservative tuning practice during the time of Bach and Handel, when new ideas and innovations were displayed to the eager for new world.

References to tuning systems that are unquestionably referring to meantone systems were published as early as 1496 (Gafurius) in *Practica musica,* and 1523 (Aron), who describes the most appropriate tuning for organs being the meantone temperament, which has every fifth tempered by 1/4 comma, or about 1/18 semitone. Consequently four fifths would produce a pure major third.[10] This temperament and its various modifications, was to be the strongest opponent of Equal temperament, at least so far as the tuning for the keyboard instruments, during the next two or three hundred years.

[10] In meantone temperament, pure thirds were favored. Previous to meantone temperament, Pythagorean tuning was primarily used where pure 5th were favored.

From the middle of the sixteenth century, all the theorists agreed that the fretted instruments, lutes and viols, were tuned in “equal temperament”, while Vicentino made the first known reference to this fact, going so far as to state that both types of instrument had been so tuned from their invention, and the fretted instruments in general had always been tuned in equal temperament.

As for the keyboard instruments, Zarlino declared, that temperament was as old as the complete chromatic keyboard and further on, Gafurius stated among the eight rules of counterpoint that “organists assert that fifths undergo a small, indefinite amount of diminution called temperament (participata).[11]

Salinas (in De musica libra septum) presents three different meantone temperaments: the 1/3 comma system (he is the likely the inventor of the 1/3 meantone system), the 1/4 comma system (the most common meantone

[11] The tuning is to be made in three successive stages.
1. First, the major third, C-E, is to be made "sonorous and just." But the fifth C-G is to be made "a little flat." Same idea for the fifth G-D, D-A and A-E.
2. In the second stage of tuning, the fifths F-C, Bb-F, and Eb-Bb are tempered exactly the same as the diatonic fifths had been.
3. Finally, in the third stage, C# and F# are tuned as pure thirds to A and D respectively. The name "meantone" was applied to this temperament because the tone, as C-D, is precisely half of the pure third, as C-E. Practically when you tune an instrument in meantone temperament, you have to start with tuning four perfect fifths upward from C, which produces a major third C-E that is wide by 21.5 cents, so in order to produce a perfect major third C-E, these four fifths are each narrowed by one fourth of 21.5 cents = 5.375 cents. Then B is tuned up a perfect major third from G, and F is tuned down a perfect major third from A, which should complete the naturals. Next step is sharps which are tuned upward a perfect third, and flats which are tuned downward a perfect third. It can be shown that tuning a G# up an perfect third is equivalent to continuing to tune up from E by using fifths each narrowed by 5.375 cents:

E - B - F# - C# - G#

Similarly, tuning an A flat down a perfect third is equivalent to continuing to tune down from C by using fifths all narrowed by 5.375 cents:

C - F - Bflat - Eflat - Aflat - Dflat

The discrepancy between an associated sharp and flat then is 21.5 (3) – 23.46 = 41 cents. Because this temperament is regular, all associated sharps and flats differ by 41 cents (sharps are 41 cents flatter than associated flats).

temperament), and the 2/7 comma system described in detail by him and Zarlino, apparently independently. Zarlino called the meantone temperament a "new temperament" and said that “it is very pleasing for all purposes when used on keyboard instruments."

(from Gioseffo Zarlino, *Dimostrationi armoniche*, Venice, 1571, p. 267)

Although meantone is best known as a tuning environment associated with the earlier music of the Renaissance and Baroque, it continued to be used as a keyboard temperament well into the middle of the 19th century. Meantone temperament has had considerable revival for early music performance in the late 20th century and in newly composed works specifically demanding meantone by composers including John Adams, György Ligeti, and Douglas Leedy. Further to my previous affirmations, I want to propose a comparison between those tuning systems.

Table 4: List of frequencies and the distance between them in cents

(comparison of Equal, Pythagorean and Meantone temperaments)

Equal Temperament

	C	C#	D	D#	E	F	F#	G	G#	A	A#	B	C
Cents	0	100	200	300	400	500	600	700	800	900	1000	1100	1200
Distance	100	100	100	100	100	100	100	100	100	100	100	100	

Meantone Temperament

	C	C#	Db	D	D#	Eb	E	F	F#	G	G#	Ab	A	A#	B	C
Cents	0	76	112	193	269	311	386	504	580	697	773	814	890	1007	1083	1200

Distance in cents	76	112	117	81	75	118	117	76	118	76	117	76	117	117	76	117	76	117
	C-C#	C-Db	C#-D	Db-D	D-D#	D-Eb	D#-E	Eb-E	E-F	F-F#	F#-G	G-G#	G-Ab	G#-A	Ab-A	A-A#	A#-B	B-C

Pythagorean Temperament

	C	C#	D	D#	E	F	F#	G	G#	A	A#	B	C
Cents	0	112	204	316	386	498	603	702	814	884	1018	1088	1200
Distance	112	92	112	70	112	105	99	112	70	134	70	112	

1.3 Regular and irregular meantone temperaments

Meantone temperaments in which the good fifths are all the same size (except for the wolf fifth), such as Aaron, Salinas, Zarlino and Silbermann's are called regular, while the irregular meantone temperaments are characterized by having more than one size of good fifths (and thus thirds)[12] such as Kirnberger, Valotti, and Werckmeister.

Typical within the regular meantone temperament range is Silbermann's (used in the high Baroque for organs) in which the fifths are tempered by 1/6 of a syntonic comma or nearly four cents. The following table shows that the good major thirds are about seven cents sharp and the good minor thirds about eleven cents flat, and that the wolves are still present though a bit mellower.

Table 5: Differences in cents between different temperaments

Temperament	Pythagoras	Salinas 1/3 comma	Aaron 1/4 comma	Silbermann 1/6 comma	Equal 1/11 comma
Fifth	0	-7.3	-5.5	-3.7	-2
Dim. sixth	-24	+56.7	+36.5	+16.3	-2
Major third	+22	-7.3	0	+7.3	+14
Dim. fourth	-2	+56.7	+42	+27.3	+14
Minor third	-22	0	-5.5	-11	-16
Aug. second	+2	-64	-47.5	+31	-16
Diff. enharm.	-24	+64	+42	+20	0

[12] Irregular meantone temperaments are characterized by having no wolf intervals to limit modulation (as in the previous temperaments except equal), and by having a more or less orderly progression in the acoustic quality of the triads.

The second group of meantone temperaments, the irregular (also known as well temperaments), which are now believed to have been very important in the past (especially during the Baroque), have generally speaking, the ditonic comma (-24 cents) distributed unevenly.[13]

1.3.1 Kirnberger, Vallotti, Werckmeister

Kirnberger's method of compensating for and closing the circle of fifths was to split the "wolf" interval in half between two different fifths. That is, to compensate for the one extra comma, he removed half a comma from two of the formerly perfect fifths in order to complete the circle. In so doing, he allowed the remaining fifths to stay pure.

Vallotti

Here, the comma is distributed equally to six consecutive fifths, those involving no raised keys, the others being pure, while the major thirds vary from not quite pure (six cents sharp) to Pythagorean, and similarly for the minor thirds.

Werckmeister

The three fifths between C and A are tempered in such a way to allow a slightly wide major sixth, plus the other tempered fifth between B and F# closing the circle. The fact that the tempered fifths are not consecutive makes this

[13] Most of it is given to the fifths of the near keys, and little, if any, to the fifths of the remote keys (in some cases, such as the French temperament ordinaire, the first fifths are tempered a bit too much, with the result that the last fifths of the circle have to be a bit sharp).

temperament less symmetric and less unequal than the Kirnberger temperament, even though the fifths are tempered essentially the same in both. More than that, the pattern of the thirds and fifths is very similar to Vallotti's temperament.

CHAPTER II

2.1 Acoustic theories of Hermann Helmholtz, Bernhard Riemann and Max Planck

In 1863 Hermann von Helmholtz, professor of physiology at the University of Heidelberg and professor of physics at the University of Berlin, launched his volume “On the sensations of tone as a Physiological Basis for the Theory of Music”, the result of a research based on the attempt to connect the boundaries of two sciences, music and acoustics. Beside his most significant developments in physics and philosophy of science in the 19th century[14], Hermann von Helmholtz (1821–1894) achieved a staggering number of scientific results and his voice was to be heard even in the domain of acoustics and music theory.

> The acoustics constantly employs conceptions and names borrowed from the theory of harmony, and speaks of the 'scale,' 'intervals,' 'consonances,' the numbers of vibrations,' and fixes their 'ratios' for the different intervals. (…) Physical knowledge may indeed have been useful for musical instrument makers, but for the development and foundation of the theory of harmony. It has hitherto been totally barren.
>
> (Helmholtz, *On the sensations of tone*, 1895, pg.27)

The German scientist and philosopher of 19th Century, raises the idea that the scientific basis of music is to be found in the properties of vibrating, inert

[14] His achievements underline the proof that Euclidean geometry does not describe the only possible visual and physical space, and marked the shift from physics based on actions between particles at a distance to the field theory.
His main scientific contributions were: the formulation of energy conservation, the vortex equations for fluid dynamics, the notion of free energy in thermodynamics, and the invention of the ophthalmoscope.
His constant interest in the epistemology of science guarantees his enduring significance for philosophy, as well.

bodies, such as strings, tuning forks, pipes, and membranes, and for the scientific development level of those times, his researches were simply revolutionary.

> The sensation of a musical tone is due to a rapid periodic motion of the sonorous body; the sensation of a noise to non-periodic motions.
> (Helmholtz, *On the sensations of tone*, *Noise and musical tone, The propagation of sound*, 1895, pg.8)

He understood that the fundamental musical tones are sine waves of various frequencies, and defined musical tones “as periodic vibrations of the air”, while every other tone is merely a superposition of added-up sine waves, called "overtones" or "harmonics." That being said, the next stunning discovery was the consonant attribute of musical intervals determined by properties of the "overtone series", which is simply called the whole-number ratios of pure sound frequencies[15].

> These are all the consonant intervals which lie within the compass of an Octave. With the exception of the minor Sixth, which is really the most imperfect of the above consonances, the ratios of their vibrational numbers are all expressed by means of the whole numbers, 1, 2, 3, 4, 5, 6… This relation of whole numbers to musical consonances was from all time looked upon as a wonderful mystery of deep significance.
> (Helmholtz, *On the sensations of tone*, pg.14, 15)

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[15] 1 : 2 Octave
2 : 3 Fifth
3 : 4 Fourth
4 : 5 major Third
5 : 6 minor Third

When the fundamental tone of a given interval is taken an Octave higher, the interval is said to be **inverted.** Thus a Fourth is an inverted Fifth, a minor Sixth ^ an inverted major Third, and a major Sixth an inverted minor Third. The corresponding ratios of the pitch numbers are consequently obtained by doubling the smaller number in the original interval. From 2 : 3 the Fifth, we thus have 3 : 4 the Fourth, 4:5 the major Third, 5:8 the minor Sixth, 5:6 the minor Third, 6 : 10= 3 : 5 the major Sixth

From this standpoint, the German scientist and researcher made the assumption that the virtue of the whole numbers has a correspondent in the purity of perfect intervals, or there is an obvious similitude in between them. Knowing that any kind of temperament is in fact, a modification or a step aside from the purity of a perfect interval, was not too difficult to "radicalize" the instrumental tuning domain.

As a result, he found well-tempering and equal-tempering were so far away from the "natural tuning" of whole-number ratios, and demanded that musicians choose the original, the natural (in some cases even the Pythagorean temperament), which acts much better on the account of the immaculacy for perfect intervals, and not necessarily on the account of frequent modulations, and not distant modulations.

An interesting comparison between Pythagorean, Well and Equal temperament is found in the chapter XVI called *The system of keys. Modulation leads to Tempering the Intonation of the Intervals* Pg. 312. Here, the author militates for the general ideal of interval purity, explaining why certain intervals are sounding better in a different instrumental temperament. For instance, on page 213 from the same writing, *On the sensation of tone*, he states that "The Thirds and Sixths of the equal temperament are nearer the perfect intervals than are the Pythagorean."

Table 6: How close are temperaments to the perfect ratios?

Intervals	Perfect		Equally tempered		Pythagorean	
	ratios	**cents**	**ratios**	**cents**	**ratios**	**cents**
Semitone	16/15	182	18/17	100	256/243	90
Minor third	6/5	316	6/5x121/122	300	6/5x80/81	294
Major third	5/4	386	5/4x127/126	400	5/4x81/80	408
Minor sixth	8/5	814	8/5x126/127	800	8/5x80/81	792
Major sixth	5/3	884	5/3x122/121	900	5/3x81/80	906

This liberal philosophy of "natural tuning" or the return to the origins of sound, owes its present power and influence in large part to the acoustical theories of Hermann Helmholtz, the nineteenth-century physicist and physiologist.

His 1863 volume, *Die Lehre von den Tonempfindungen als physiologische Grundlage für die Theorie der Musik* (The Theory of the Sensations of Tone as a Foundation of Music Theory) became, generally speaking, one of the standard reference works on the scientific bases of acoustic and music, and remains relevant in understanding different stages of tuning advancement in the history of music.

Despite this acclaimed reputation, his postulates generated a fervent scientific argument and controversy in his time, and continued from that point on. One of the reasons for that amplification process would be the geometric progression of scientific development of acoustics and physics in the 20th Century.

Nowadays, we understand a sound wave as an electromagnetic process involving the rapid assembly and disassembly of geometrical configurations of molecules, not only a “vibration of the air”. The notion of “soliton”[16] or the "Wave of Translation" in modern physics, demonstrates the moving of sound waves through the air at a constant velocity. Although much more detailed experimental work needs to be done, we know in principle that different frequencies of coherent solitons correspond to distinct geometries on the microscopic or quantum level of organization of the process. Moreover, any attempt of temperament in human civilization we know, was a step up.

This was already indicated by the work of Helmholtz's contemporary, Bernhard Riemann, who refuted most of the acoustic results of his colleague mathematician in his 1859 paper on acoustical shock waves. He tried to explain for example, from a modern point of view, why the sense of human hearing is so accurate.

He therefore looked to the physical properties of the parts constituting the human hearing organ which could be the scientific proof of its high level of performance.

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[16] It was first described in 1834 by John Scott Russell (1808–1882) who observed a solitary wave in the Union Canal in Scotland, and ulterior he reproduced the phenomenon in a wave tank.

Describing the experiments that brought to the attention the high sensitivity displayed by the ear in detecting sounds, he inquired into which means made it possible to transform a sound wave into the refined perception of Die Klang, while they keep intact the harmonic characteristics, now with standing they had to amplify the sound strength scattered in the space.

It was in this context that Riemann carried out acoustical experiments concerning the tempered tuning in comparison with “natural”, “pure” or “just” scale. He tried to explain why musicians prefer the tempered versions to the detriment of the “natural.”

The difference of opinions between Helmholtz and Riemann attracted the attention of the scientific world over the years. Another name concerning the tempered versus natural tuning, is Max Planck (1858 -1947), the German theoretical physicist and discoverer of quantum energy, laureate of Nobel Prize in Physics (1918). Planck declares in one of his music and acoustic inclusions:

> One can straight adjust his hearing to a higher or lesser degree of adaptability. The ear of a person who follows with a great attention a concert is much more adaptive when this person is attending the concert for pleasure or to act as a critic (…) To me, this moment of transition (toward natural), is an immediate source of artistic pleasure (…) The adaptation is achieved the easier and can be driven the furthest, the less the interval is consonant…This clearly shows how our ear gets used with the tempered tuning.
>
> (Max Planck, in *Die naturliche Stimmung in der modern Vocalmusik,* Pg. 423)

In addition to the many dilemmas that a conductor faces when performing a musical piece, Planck enumerated the tuning aspect, but he did not offer any solution. He gave the last word to the composer, and when it was impossible to get, he referred to the artistic effect one wished.

> Because art finds its justification in itself, and no theoretical system of music, even if it was logically founded, and developed in a consistent way, is in the position to fulfill all the requirements of art which is in continuous exchange with the human spirit. In this "spirit", the "natural" has indeed no advantage to the tempered one.
>
> (Max Planck, in *Die naturliche Stimmung*, 1894, Pg. 424)

Later, in his scientific autobiography, he remembers:

> I had the task to study the "natural" tuning on this instrument (Elitz's harmonium). I did this with great interest, in particular in relation to the issue about the role played by "natural" tuning in our modern vocal music, without instruments. In this process I obtained the unexpected result that our ear prefers the tempered tuning to the "natural" one in any circumstance. Even in a chord with harmony in a major tonality, the "natural" third sounds slack and without expression in comparison with the tempered one. Without doubt, this fact goes back to habits which have been developed over many years and generations.
>
> (Max Planck, *Die naturliche Stimmung*, 1894, Pg. 383-384)

From Helmholtz who had made "natural" Zarlino's scale by beats and overtones and advocated a return to perfect fractions and whole numbers, to Riemann who sustained the geometries on the microscopic or quantum level of solitons organization, and finally to Planck who inclined to the musicians' perspective and sensorial experiences in detriment of pure mathematics, we encounter a vast exposure of a complex process. From the genesis "purity" of no temperaments, to the well, equal or even microtonal tunings, where should we set our preferences?

CHAPTER III

3.1 Temperament and Tuning during Bach's era

In his article "Well tempering based on Werckmeister Definition", Johan Broekaert notifies us about the existence and the actual use of a multitude of historical temperaments during the life of J. S. Bach, as well as the different tuning categories of that time. Bach himself introduce one of these systems in the title of his 1722 collection amongst a large variety of non-equal temperament systems which were then in common practice.

Over centuries, up to the present, musical requirements led to more than one hundred historical musical temperaments, while most composers or theoreticians were involved in acoustic and temperament studies.

Music composition pioneers were often artistic and technical trend openers with the decisions of "music and temperament making" in their own hands. It is challenging to think of the masters of baroque composition as persons who would use temperaments designed by other researchers, especially in an époque of an unprecedented development in all domains. It seems probable that tuning decisions were made following the charts of Werckmeister's *Musicalische Temperatur*.

First of all, attempts to reconstruct Bach's temperaments were made by a number of musicologists, from Kirnberger and Marpurg in the 18th century to

20th-century musicologists Herbert Kellner and John Barnes, and 21st century Bradley Lehman.

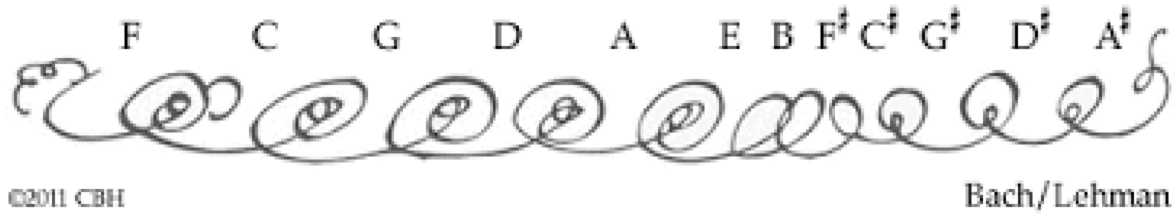
In the February and May 2005 issues of *Early Music* magazine, Lehman, the harpsichordist and mathematician, referred to 21 temperaments, claimed to be Bach-temperaments. While examining the title page of the Well-Tempered Clavier from 1722, Lehman noticed that the decorative scroll above the text features 11 loops of three different kinds (simple, double, and convoluted).

Figure 1: The title page of Bach's Well-Tempered Clavier from 1722, with labels added by the author of this article

(from Tamar Halperin, 2009, The Ongoing Quest for Bach's Temperament in The Juilliard Journal, https://www.juilliard.edu/journal/ongoing-quest-bachs-temperament)

He observed that 11 is the number that would describe the temperament of 12 fifths (if the first note is given, it would also be the last note of the circle, rendering the “12th loop”). It occurred to him that the letter C is attached to the first loop from the right, and then he decided to turn the loops upside down for a better prospective.

Figure 2: Lehman’s interpretation of the scroll; the same graphic as in Fig. 1, turned upside down, with note names above their respective loops



From Tamar Halperin, 2009, The Ongoing Quest for Bach's Temperament in The Juilliard Journal, https://www.juilliard.edu/journal/ongoing-quest-bachs-temperament

The researcher states that in the Baroque period the normal amount of tempering a fifth was 1/6th of a Pythagorean comma, which, he observed that is represented in the squiggle by the convoluted spiral (Figure 1). The next step up was to interpret the double-spiraled loops representing the tempering of 1/12th of a Pythagorean comma, and the simple loops to indicate pure fifths.

The result of this analysis was a well temperament supposedly the one (or one of the them) used by Bach. In Lehman’sopinion, it brings out qualities of Bach’s composition that are hidden in Equal temperament.

> Well temperament means a mathematical-acoustic and musical-practical organization of the tone system within the twelve steps of an octave, so that impeccable performance in all tonalities is enabled, based on the extended just intonation (natural harmonic tone system), while striving to keep the diatonic intervals as pure as possible. This temperament acts, while tied to given pitch ratios, as a thriftily tempered smoothing and extension of the meantone, as unequally beating half tones and as equal (equally beating) temperament. (Andreas Werckmeister, *Orgelprobe*, 1681)

For the first time in music history, the terms "well-tempered" and "well-temperament" were created in 1691 by German theorist Andreas Werckmeister, referring to the temperament in which the fifths are of different sizes, but none of the fifths is too false to be easily noticed as a "reason for acoustic discomfort."

In those years, the meantone was widely used and considering its endurance (around 400 years, from the late 15th century, all the way through the 19th century), one can conclude that it was the most successful tuning before the invention of the equal temperament.

The generating principle behind meantone temperament was embedded in the primacy of clean thirds, in other words, it was more important to preserve the consonance of the major thirds than it was to preserve the purity of the perfect fifths. There are acoustical and theoretical reasons for this, namely that the thirds and fifths belong to different consonance categories, perfect and imperfect.[17] The

[17] It should also be noted that the concepts "consonance" and "dissonance" are highly context-related. The way **sonance** factor is perceived, depends on several music-psychological factors: temperament, genre (in atonal music, consonances are scarce), timbre, the exposure to consonant or dissonant music, or the extent of the interval (can be several octaves), and so on.
More than that, the difference between perfect and imperfect consonance is explained by the ratios and common harmonics.
For example, Pythagoras thought that two tones are consonant, when their ratio consists of whole numbers with an even division: consonant: octave 1:2, 5th 2:3, 4th 3:4, maj.3rd 4:5, while dissonant: maj.2nd 8:9, maj.7th 8:

notes in a slightly out-of-tune third, being closer together than those in a fifth, create faster and more disturbing beats than those in a slightly out-of-tune fifth.

The aesthetic motivation for meantone temperament was that composers and their audiences found the clarity and the "acoustic logic" of the major third, and their generation felt the responsibility to complete the relevant acoustic spectrum for a humanity in a full scientific and artistic progress. All these endeavors not because the medieval austerity of open perfect fifths and octaves are not representing viable artistic ideals, but in fact, as a historical attempt to add the relevant portion to the panoramic picture, meanwhile respectfully conserving what previous generations discovered.

The following is a chart of a ¼ comma meantone temperament defined in 1523 by Pietro Aaron.

Table 7: ¼ comma Meantone temperament

Pitch:	C	C#	D	Eb	E	F	F#	G	G#	A	A#	B	C
Cents:	0	76.0	193.2	310.3	386.3	503.4	579.5	696.8	772.6	889.7	1006.8	1082.9	1200

15. The complicated ratios were seen as a sign of imperfection (for example the tempered 5th with 293:439 ratio), and the even and clean ratio, an "acoustic image" of perfection (eg. octave 1:2)
Hermann von Helmholtz, the great German physician and physicist of 19th Century, thought that tones are consonant or perfectly consonant, when one or more of their harmonics fall together, if the frequency values have common multiples.

Table 8: The sizes of the major thirds and perfect fifths on each pitch

Major third	Cents	Perfect fifth	Cents
C-E	386.3	C-G	696.8
Db-F	427.4	Db-Ab	696.6
D-F#	386.3	D-A	696.5
Eb-G	386.5	Eb-Bb	696.5
E-G#	386.3	E-B	696.6
F-A	386.3	F-C	696.6
F#-A#	427.3	F#-C#	696.5
G-B	386.1	G-D	696.4
Ab-C	427.4	Ab-Eb	737.7
A-C#	386.3	A-E	696.6
Bb-D	386.4	Bb-F	696.6
B-D#	427.4	B-F#	696.6

Unlike meantone temperament, in well-temperament the fifths are smaller than pure fifths (some of them and scarce, bigger), which makes all the tonalities being playable, yet they vary in their purity and timbre. The different "colors" of the various tonalities enhanced by non-equal temperament were considered an advantage and this is conducting our thoughts to the concept of variety, an important part of Baroque aesthetics.

While the Pythagorean tuning includes eleven pure fifths, and a reduced fifth, that can be fixed on G#-D# (or Eb), well temperaments requires the best possible purity for fifths and thirds and keep the diatonic intervals as pure as possible. The same idea of even division, for example of the Pythagorean comma over the twelve fifths, leads to fifths that all become slightly reduced, and quite a gain in purity for major thirds that were Pythagorean before. As a general outcome, the over-all impurity and acoustic beating phenomenon, has been reduced to a limit of minimum, fact that is witnessing for a big step up toward equal temperament.

3.2 Variations of Werckmeister well temperament

3.2.1 Werckmeister I or the "correct temperament" - based on 1/4 comma divisions

Similar to the purity of fifths of the Pythagorean tuning, this temperament uses mostly perfect fifths, only four of them (C-G, G-D, D-A and B-F♯) are tempered by 1/4 comma. Based on the acceptable level of consonance of all tonalities, Werckmeister designated this tuning as particularly suited for playing chromatic music. These clear indications contributed to its popularity as a tuning for J.S. Bach's music in recent years, and it could be just the beginning for a different approach and prospective.

3.2.2 Werckmeister II - based on 1/3 comma divisions

In Werckmeister II the fifths C-G, D-A, E-B, F♯-C♯, and B♭-F are tempered narrow by 1/3 comma, and the fifths G♯-D♯ and E♭-B♭ are widened by 1/3 comma, while the other fifths are pure (most of its intervals are close to sixth-comma meantone temperament). Werckmeister himself organized this tuning for playing mainly diatonic music and rarely or never use accidentals.

3.2.3 Werckmeister III - additional temperament based on 1/4 comma divisions

In Werckmeister III the fifths D-A, A-E, F♯-C♯, C♯-G♯, and F-C are narrowed by 1/4, and the fifth G♯-D♯ is widened by 1/4 comma, meanwhile the other fifths are pure. The number of tempered intervals and the amount in cents for each modification, leads to the conclusion that it is closer to Equal temperament than the previous two.

Table 9: The frequency chart for Werckmeister III

Pitch:	C	C#	D	Eb	E	F	F#	G	G#	A	A#	B	C
Cents:	0	90.225	192.18	294.135	390.225	498.045	588.27	696.09	792.18	888.27	996.09	1092.18	1200

It is noticeable that no pitch is more than 12 cents off when compared with Equal temperament. The following perfect fifths are 3/2 ratios of 701.955 cents each: Gb - Db - Ab - Eb - Bb - F - C, as well as A - E - B. The Pythagorean comma is distributed among the remaining fifths, C - G - D - A and B - F#, each of which is 696.09 cents.

Table 10: The sizes of the major thirds and perfect fifths on each pitch (organized by the circle of fifths)

Major third	Cents	Perfect fifth	Cents
C-E	390.225	C-G	696.09
G-B	396.09	G-D	696.09
D-F#	396.09	D-A	696.09
A-C#	401.955	A-E	701.955
E-G#	401.955	E-B	701.955
B-D#	401.955	B-F#	696.09
F#-A#	407.82	F#-C#	701.955
Db-F	407.82	Db-Ab	701.955
Ab-C	407.82	Ab-Eb	701.955
Eb-G	401.955	Eb-Bb	701.955
Bb-D	396.09	Bb-F	701.955
F-A	390.225	F-C	701.955

3.2.4 Werckmeister IV (VI) the Septenarius temperaments

It is based on a division of the monochord length into 196=7x7x4 parts where the various notes are defined by which 196-division one should place the bridge on in order to produce their pitches. Mathematically speaking, it is quite logical and distinct from the “irrational” tempered values of the other well variations, while in practice, both involve pure and impure sounding thirds and fifths. Werckmeister described the Septenarius as "an additional temperament which has nothing at all to do with the divisions of the comma, nevertheless in practice so correct that one can be really satisfied with it."

Septenarius temperament has the fifths C-G, G-D, D-A, B-F♯, F♯-C♯, and B♭-F tempered narrow, while the fifth G♯-D♯ is tempered wider than pure and the other fifths are perfect.

Werckmeister addressed his readers as fully professional, and he carefully avoided to indicate that he could teach them anything at all. He said, in the *Musicalische Temperatur* near the end of the foreword,

> Just as it was not my intention in my *Musicalischen Wegweiser* to prescribe anything to any outstanding *Musico*, inasmuch as I find myself much too humble for that, and would commit a huge mistake: Similarly, in the present *Tractat* no experienced *Musico* will be burdened with how to tune a tempered keyboard instrument.

A little later, Werckmeister continues:

> In this book, I demonstrate to those who are eager to learn it, how the temperaments can be formulated and arranged in various ways. One may place the beatings of the fifths in whichever keys one wants; it is just that the perfect consonants should not be treated too much. It is enough when a keyboard is so tempered that it is usable throughout (that is, only as many fifths should be tempered as needed for the instrument to be playable in all keys).
>
> (Andreas Werkmeister, *Musicalische Temperatur*,1686, Pg. 57-58)

Conclusion

It appears that Andreas Werckmeister did not have the intention to inforce any particular tuning upon his contemporaries. Instead, they were granted the freedom to develop well temperaments of their own, and not just use the ones that he had published.

CHAPTER IV

4.1 Equal Temperament

From Meantone Temperament to Equal Temperament

The whole music industry began to switch to equal temperament at the end of the 18th century, one of the reasons being the Industrial Revolution, the time when many instruments were redesigned, and standardized, even though the piano, the harp, and the organ had switched to well temperament at an earlier point because it was difficult to re-tune those instruments. Wind instruments and brass instruments, which had previously been flexible enough to adjust pitch as the music required, were changed; "instruments were standardized to play a chromatic scale such that the ‘centers’ of their pitches corresponded as closely as possible to the pitches of twelve-tone equal temperament."[18]

Orchestras became common, and there had to be standardization between the orchestral instruments, pianos being used as instruments in the orchestras, as soloists, and since they were tuned already in an equal temperament (and so did the other instruments), they were the predominant instrument for musical training. Musicians began to be trained in equal temperament, as opposed to meantone temperament, and that meant if composers wanted to hear their works performed, they had to expect the music to be performed in equal temperament. The change to

[18] Doty, David B. The Just Intonation Primer. "The Just Intonation Network". URL: http://www.dnai.com/~jinetwk/primer2.html (26 Apr. 1997)

equal temperament happened at different times in different countries, but it is thought to have first started in France and Germany in the last quarter of the 17th century with keyboard instruments.

> The equal temperament came into use in Germany before it was introduced into France. In the second volume of Matheson's Critica Jusica, which appeared in 1752, he mentions Xeidhard and Werckmeister as the inventors of this temperament, Johann Sebastian Bach had already used it for the clavichord {clavier), as we must conclude from Marpurg's report of Kirnberger's assertion, that when he was a pupil of the elder Bach he had been made to tune all the major Thirds too sharp. Sebastian's son, Emanuel, who was a celebrated pianist, and published in 1753 a work of great authority in its day 'on the true art of playing the clavier,' requires this instrument to be always tuned in the equal temperament.
>
> (Helmholtz, *On the sensations of tone*,1895, Pg.322)

Meantone temperament was the common system in France until the mid-18th century, and England still used meantone temperament until the mid-19th century. Like the systems of Agricola in the sixteenth century and of Dowland in the early seventeenth century, many of the numerous irregular systems of the eighteenth century contained more pure than impure fifths. The instruments of the violin family, tuned by fifths, have a strong tendency toward the Pythagorean tuning.[19]

[19] There is an interesting fact materialized in an question: At what extend could a pupil studying violin hearing different than the other studying piano, so being exposed from very beginning to more pure fifths. And from this point, another question arise: It is equal temperament something what we inherit “genetically” from our parents, or we develop that getting in contact with musics and instruments using it?!

The history of equal temperament continues in Middle Age with two German methods of organ temperament. The earlier of the two was Arnold Schlick's temperament, an irregular method for which his directions were somewhat vague, but in which there were ten flattened and two raised fifths, as well as twelve raised thirds. Actually, from Schlick's own account, the method lay somewhere between the meantone temperament and the equal temperament. More definite and certainly very near to equal temperament was Grammateus' method, in which the white keys were in the Pythagorean tuning and the black keys were precisely halfway between the pairs of adjoining white keys.

The first precise mathematical definition of equal temperament was given by Salinas:

> We judge this one thing must be observed by makers of viols, so that the placing of the frets may be made regular, namely that the octave must be divided into twelve parts equally proportional, which twelve will be the equal semitones.
>
> (Francisco Salinas, *De musica libri*, 1577, Pg. 173)

On the continuum for the history of temperament, he first tuning rules that might be interpreted as Equal temperament were expressed by Giovanni Maria Lanfranco, in his *Scintille de musica* (Brescia, 1533, p. 132), and they were for clavichords and organs (Monochordi & Organi), eventually extended for fretted instruments. Lanfranco's essential rules concern the tempering of the fifths and the thirds:

> The fifths are to be tuned so flat "that the ear is not well pleased with them, and the thirds as sharp as can be endured.
> The enlargement of the major third, the diminution of the minor third, the equivalence of the notes C# and Db , F# and Gb—these are essential departures from his contemporaries.
>
> (Otto Kinkeldey, *Orgel und Klavier in der Musik des 16 Jahrhuderts*, 1910, Pg.77)

> For keyboard instruments, Lodovico Zacconi recommended Aron’s meantone temperament, while "for the other instruments, such as the viola da braccio, viola da gamba, violins, and others, he indicates clearly how each one is to be tuned.
>
> (Lodovico Zacconi, *Prattica di musica*, Venice, 1592, Pg. 218)

> In Zacconi's day and long before it, the fretted instruments were said to have equal semitones. To Zarlino, Salinas, and Galilei this meant equal temperament, with all semitones equal. To Grammateus and Bermudo, only ten semitones were equal, the others being smaller; to Artusi, and presumably also to Bottrigari and Cerone, there were ten equal semitones, the other two being larger. But, of these three types of temperament equal, modified Pythagorean, and modified meantone— only equal temperament had both flat fifths and sharp thirds in addition to equal semitones.
>
> (Murray J. Barbour, *Tuning and Temperament, Equal temperament*, 1951, Pg. 46)

Taking into consideration the excellent tuning methods of Lanfranco's immediate predecessors, Grammateus and Schlick, it is conceivable that Lanfranco had the idea of equal temperament for all instruments of a full Baroque symphony orchestra of his time.

Another name to be considered, taking into account his largely circulating tuning rules, was Godfrey Keller, the German keyboard player and composer active like Handel, in England. His tuning concerns were basically concentrated on the amount of tempering the fifths of harpsichords or spinets, and he affirms that they should be as flat as the ear will permit.

> Barthold Fritz gave tuning rules for equal temperament[20] that merited the approval of Emanuel Bach, to whom he had dedicated his little book. Emmanuel Bach said that "in his few pages everything had been said that was necessary and possible, and that would satisfy far more needs than the sundry computations with which many a man has racked his brains"
>
> (Murray J. Barbour, *Tuning and temperament, Equal temperament*, 1951, Pg. 47)

> Hence older musicians especially recommended the equal temperament for the pianoforte alone. Matheson, in doing so, acknowledges that for organs Silberman's unequal temperament, in which the usual keys were kept pure, is more advantageous.
> Emanuel Bach says that a correctly tuned pianoforte has the most perfect intonation of all instruments, which in the above sense is correct.
> The great diffusion and convenience of pianofortes made it subsequently the chief instrument for the study of music and its intonation the pattern for that of all other instruments.
>
> (Helmholtz, *On the sensations of tone*, 1895, Pg.323)

4.2 Historical temperaments now and then

For the last century, the topic of temperament has been relegated to the "tall weeds" in the field of musical discussion, while the supremacy of equal temperament is world recognized and appreciated. However, fairly recent researches now strongly indicates that modern tuning is quite different from that used in most composers' time, and as a consequence, it is easy to imagine for

[20] Barthold Fritz, *Anweisung wie man Claviere, Clavicins, und Orgeln, nach einer mechanischen Art, in alien zwblf Tonen gleich rein stimmen konne*, Leipzig, 1780

example, a Beethoven piano sonata played in Equal temperament, which sound fundamentally different from the same music played in a temperament of his period, regardless of whether the instrument used is a fortepiano or a modern concert grand piano.

Instrument use with the "colors" of Well temperament and the music of a bygone era that was created with them, produces dazzling effects in the modulatory passages; the flowing changes of musical tension are enhanced by the contrast of dissonance and consonance, musical effects that the composers certainly were aware of, effects that could be enhanced in historical temperaments and faded in Equal temperament.

The "acceptable" amount of tempering has changed over the course of history when composers and audiences have demanded and created different style of music for different occasions, at different times. For instance, in the meantone era, when music experienced an intricate yet relevant influence from theology and politics[21], a good third or fifth was a pure third or fifth, and everybody knew where the wolf lurked. On the opposite, today, with the ubiquitous use of Equal temperament, we have come to accept the diaphanous and dimming exposure of pure intervals and contrast between the tonalities, with the sensible understanding

[21] Theology and politics as a unibody leading organism of a certain society as expressed in detail by Raymond Plant in his “Politics, Theology and History”, Cambridge University Press, 2001

that we have deprived ourselves in a way, of some depth and meaning that was conveyed by the masters through their mastery.

Starting with the original Pythagorean temperaments and Just Intonation of later tuning history, where the purity of fifths and octaves tried to express something from the purity of the unique universal ideals of truth and beauty, and continuing with the Well temperaments of the Classical Era, created during a time when art, science and religion diversified and expanded toward new limits, historical tunings mark a prime and substantial cultural exponent of different historical periods.

The trend of instrument tunings advanced from purity and extreme dissonance (Pythagorean, Just Intonation), passing through the mixture of pure and tempered intervals, where we hear the "calm and the storm" (meantone temperaments, well temperaments), and finally, resting on the field of perfect balance and beneficial accommodations for all the instruments of the symphony orchestra (equal temperament).

CHAPTER V

5.1 Other tunings and digital technologies

As the research and evolution of tuning and temperament progressed, a certain stability was created once equal temperament controlled more and more musical terrain. Musical concepts like harmony, rhythm, tempo, dynamics, structure, and sound emission, to mention just a few, have been influenced and ultimately developed to such a fluid and systematic tonal base theory. Even though the tonal system is vast and detailed, the need for more creative musical outlets has guided composers and theorists to create sophisticate landscapes of sound using different systems, for example microtonality and atonalism. In this chapter I want to outline microtonal tunings being a fundamental part when managing microtonality.

5.2 Creative Approaches to Temperament and Tuning

As a professionally active piano tuner and academically trained organist I have always had an intense interest in the art, history, and science of instrument tuning methods, tuning systems, historical tuning trends and schemes, alternative tunings, and the potential of new tuning systems to influence the creative process of composition. This interest involves the application of pure academic research in the form of compositions in various alternate tunings, micro-tunings, as well as the creation of original tuning systems.

The challenge of moving beyond fixed pitch instruments, such as the piano and organ, towards a theoretical approach to temperament and tuning which can be made manifest by digital and electronic methods. This approach will enable creative analysis and experiments related to instrumental tuning. As is observed in the unfolding of different periods in music history, every homogenous music époque and related form of artistic expression, had its own methods and instruments to formulate and articulate the leading ideals and the new directions. Almost all historical music “eras” advanced unique tuning systems, or at minimum displayed several possible systems as viable artistic options.

5.3 The origins of microtonality and microtonal tuning

Antiquity is perhaps best defined by the Pythagorean experiments and system and a great array of research in tuning temperaments by scholars such as Aristoxenus, Claudius Ptolemy, and Didymus. The middle ages could be represented by a continuation of different Pythagorean tuning variations, but starting with Renaissance period, the theoretical and practical solutions were created and thrived in various different locations. If Renaissance musicians revived and developed tuning models and temperaments, then general interest in the area was more completely advanced during the Baroque. The number of theorists, treatises, and practitioners interested in tuning as a means towards artistic truth and expression is considerable.

Over many centuries, from the ancient Greeks to modern times, a rich and complex line of research, musical exploration, and artistic speculation led to more than one hundred unique musical temperaments. Numerous composers and theoreticians have involved themselves in acoustic and temperament studies in pursuit of artistic excellence. (e.g., Pythagorus, Aristoxenus, Boethius, Aaron, Ramos, Zarlino, Vicentino, Werckmeister, Valotti and Young, Kirnberger, Marpurg, Barnes, Stockhausen, Carlos, Partch, etc.)

Many of the pioneers of western music composition were artistic and technical trend setters and some made influential decisions regarding “music and specific temperaments.” It is challenging to think of earlier composers using temperaments designed by other researchers, especially during époques of unprecedented development in various domains.

It seems likely that “personal” tuning decisions, perhaps with the aid of charts similar to Werckmeister’s *Musicalische Temperatur* as a guide, were made as part of the compositional process. This idea, linking or re-linking compositional process and decisions to different temperaments forms the basis of my proposal.

A very basic survey of some limited aspects of the world of alternate tunings (e.g., non-Equal temperament) follows and offers some context for the complexity and creative potential of this research.

Moreover, I want to pinpoint another segment of great interest for this research, which offers a complete and spectacular prospective. Micro-tuning or micro-tonal tuning systems were used from antiquity[22], especially the Hellenic civilizations of ancient Greece, which recognized three genera of tetrachords: the enharmonic, the chromatic, and the diatonic. Knowing that the intervals were of many different sizes, microtones too, the enharmonic tetrachord featured notably intervals of a distinctly "microtonal" nature and not necessarily equal to half of a semitone, 50 cents, where a contemporary semitone is 100 cents.

Later on, in the Renaissance era, the Italian composer and theorist Nicola Vicentino (1511–1576) worked with microtonal intervals and built a keyboard with 36 keys to the octave known as the archicembalo. The main exception from a contemporary keyboard is that the sharp keys are divided in two sections, one for a flatter accidental and one for the sharper variant.

Using the archicembalo, it was possible to play acoustically satisfactory intervals in any key, and therefore some of the innovative music composed in a chromatic style, which was only in tune when was sung by a vocal group, could

[22] We have the most music information about Greek antiquity, but by extrapolation one can conclude that microtonality was largely used in ancient times by all or most civilizations.

of be played on the keyboard, as well. The Italian theorist divided the octave into 31 equal parts, achieving a good intonation for the thirds and sixths but dealing somewhat with the acoustic beats of narrow fifths.

> While theoretically an interpretation of ancient Greek tetrachordal theory, in effect Vicentino presented a circulating system of quarter-comma meantone, maintaining major thirds tuned in just intonation in all keys
> (Murray J. Barbour, Tuning and Temperament, 1951, Pg.117-118)

Later on, along with other composers, Claude Debussy was one of the prominent names who had a great influence for the spread of the new – old micro-tuning spectrum. Musicology writings have ascribed Debussy's subsequent innovative use of the whole-tone (six equal pitches per octave) tuning in such compositions as the *Fantaisie for piano and orchestra* and the “Toccata” from the suite *Pour le piano.*

The basic interest in the microtonal intervals found between the higher frequencies of the overtone series is the compositional foundation of Debussy's works like *L'isle joyeuse, La cathédrale engloutie, Prélude à l'après-midi d'un faune, La mer, Pagodes, Danseuses de Delphes,* and *Cloches à travers les feuilles*.

In modernism, electronic instruments facilitate the use of any kind of microtonal tuning and allow “gallivanting with micro steps” at all times. In 1954, Karlheinz Stockhausen built his electronic *Studie II* on an 81-step scale starting from 100 Hz with the interval of 51/25 between steps, and in *Gesang der Jünglinge* (1955–56) he used various scales, ranging from seven up to sixty equal divisions of the octave.

In the United States, Wendy Carlos as well, experimented microtonal systems including just intonation, and used alternate tuning scales combining music from old world cultures with new musical ideas. Upon graduation from Columbia University where she earned a MA in Composition in 1965, she worked as a recording engineer and befriended Robert Moog, thus helping him by promoting the popularization of the Moog modular synthesizer. Her composition career began with 1968 album *Switched-On Bach* and almost immediately the recording acheived platinum sales status, which propelled the Moog synthesizer into the public consciousness. The album won three Grammy Awards.

In 1969 Carlos refined her techniques in *The Well-Tempered Synthesizer* album, and introduced the use of vocoders for synthesized singing in her score for Stanley Kubrick's film, *A Clockwork Orange*. This was many years before the synthetic voices became common in cinematography music.

The continuous blend between symphonic orchestra and digital and analog synthesizers, an often imitated combination, was one of her main attraction as a composer. *Digital Moonscapes* album (1984) introduced the "LSI Philharmonic Orchestra," a digital replica of orchestral timbres virtually indistinguishable from their acoustic instrumental counterparts.

Another important 20th Century artist was Harry Partch whose experimental ideas were realised in his work as composer, music theorist, and creator of musical instruments. Partch divided the octave into 43 unequal tones derived from the natural harmonic series. This "corporeal music" was one way he described his music and distinguished it from abstract or astral, which he envisioned as the dominant direction in Western music since the time of Bach. If every era and culture in the universe of world music is marked by a specific or unique tuning systems, why not undertake historical and theoretical research which informs our understanding and enables future artists to more easily to select artistically appropriate systems for musical creations which are esthetically pleasing and able to express the best of musical culture?

Conclusion

Musical tuning and temperament changed (standardizations, both of pitch and of temperament) from Antiquity passing the Middle Ages, Baroque, Classic and Romantic Period. This phenomenon strongly demonstrates that instrument tuning marks an important step up in human civilization being one of the first attempts to incipient universal culture and human development.

More than that, historical tunings represent the decisive tool for understanding the basic principles of tuning and the interesting journey for the equal temperament, being the perfect indicator for measuring "times and ages".

Since many composition circles nowadays are leaning towards a return to historical temperaments, and many scientific research projects on all continents are fostered to enhance and rejuvenate the bygone musical atmosphere, is not too difficult to infer that this is likely what will occur in the next period of time.

Through the interest for tuning and temperament, we can comprehend the human priorities at certain points in our civilization, we can perceive the ancestral aspiration for perfection at work, and we can see the limits of our exquisiteness, and we can touch with our human nature, the infinite and the absolute.

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